This study provides a restatement and extension of May's Theorem, on simple majority rule, with two alternatives and a binary agenda, a tournament structure or choice function. The conditions specified for May's Theorem are having 1) a single valued, defined, group decision function, implementing constraints with 2) balloting neutrality between the alternatives, 3) anonymity of individual votes cast, and 4) a positive association in numbers of voters and votes cast or counted. The first condition actually represents two conditions on a function, such as a group decision mechanism, a voting rule for making collective judgements, or some type of a social choice rule, function, or relation. The fourth condition is a density of votes' requirement that can be generalized to models of voter participation and election administration. Assuming conditions two and three may be considered necessary for fair and open voting in elections for representatives.

The sufficiency of May's conditions for simple majority rule (SMR) can be determined from Kakutani's fixed point theorem, by specification of a single-valued, uniquely defined function. For a closed and bounded and therefore compact set of alternatives, these conditions result in a uniquely defined median in the voting space. A binary agenda, may therefore select a voting median that is preferred to all others under SMR. On this basis a Condorcet winning alternative exists, and it is equal to a voting median. By sufficiency, a voting equilibrium exists and it represents a grand median, that is selected with either a binary (Condorcet) agenda or choice function (SMR). In paired comparisons, a voting majority reveals preferences for one or the other alternative, where the location of the voting median may be deliberated for implementing an alternative or for determining a quorum for the purposes of establishing jurisdiction

The balloting condition is a generic requirement for voting and therefore a complete vote space. Balloting describes agenda setting, by design of the organization of alternatives for the electorate and outcomes in the vote space constituting electoral returns. Any balloting condition may be considered a technical requirement *of* voting such that any voting rule or electoral setting requires that votes are cast for an election to be held. If and when votes are cast, this requires some form of a ballot to organize the vote for individual's, in addition to other necessities of election administration. On an election night, after the poling stations are closed, votes cast in regular-geographic and absentee ballot precincts are counted to produce election returns. The vote space(s) may therefore be defined in terms of the allocation of individual votes by voter registration and the distribution of votes for alternatives reported as election returns. The importance of election administration to voting implies a balloting condition prerequisite for any vote space defined by election administration.

This study provides an analysis of the original statement of May's four conditions for simple majority rule. The original conditions define a decision function with three constraints. The results describe necessary and sufficient conditions for simple majority rule (SMR). The basic results demonstrate a general equilibrium exists for any single-valued social or group decision function. Additional results demonstrate the necessity of a balloting condition to adopt and implement the method of majority rule/decision.

Revised statements of May's Theorem incorporate varying constraints by substitution for the original four conditions. These substitutions include replacement of any requirement for a group decision function that is assumes a single-valued correspondence. The results for the revised statements emphasize the decisiveness constraint to produce simple majority rule.

The balloting condition is derived from local politics that implies the ballot form structures the sequence of voting. Because ballot forms vary, this result implies outcomes in the vote space are produced by ballot form. As a consequence, vote space outcomes are structure-induced equilibrium derived from ballot forms. The ballot form may influence such matters as voter turnout and the relative numbers of votes allocated to various alternatives. By organizing the alternatives for a vote, the form of the ballot determines vote choices and therefore introduces the possibility of manipulation of outcomes in the vote space. This possibility may be seemingly less likely, with a small number of structured alternatives, such as those decisions separating voting by political party from voting by office or position on the ballot. Even using short-ballots, other possibilities exist voting by constituency seat or position versus selection from a partisan list of candidates.

Balloting refers to voter's casting ballot forms by polling and then having their votes tallied or counted by election administration. An election is constructed from the distribution of votes for alternatives tallied or counted and reported by government (election) administration. Balloting describes vote tallies or counts by voting alternative, with the final vote considered a reportable result for the public purposes of election returns. In this sense, balloting refers to voting and ongoing counting or tallying of votes required to determine varying support levels for alternatives. Repeated votes, such as those structured in rounds of voting, produce selection of hierarchical subsets of alternatives, resulting in a reduction in the number of alternatives. By doing so, balloting is a process of reducing the number of alternatives to two, for majority rule comparison, and then selection of a winning alternative. Balloting may only reduce the number of alternatives to a small number, allowing for the use of other voting rules and procedures.

The balloting condition implies May's conditions for simple majority rule. Even with substitutions of conditions, balloting is a necessary condition for various requirements of simple majority rule. Balloting not only generates outcomes in the voting space, but also provides an explanation for why various conditions are stipulated for the purposes of adoption and implementation of simple majority rule. The balloting condition results demonstrate the importance of election administration to the relationship between vote choice and electoral outcomes in the vote space. The results suggest election administration is necessary to guarantee the existence of democratic methods of majority rule and decision. The results also reveal the potential for strategic manipulation by election administration related to such factors as voter registration, ballot forms and local government structures, early and absentee voting, the location and number of precincts or polling stations, resource capacity for counting or tallying votes, recounting rules and requirements, and any provisions for validation of election returns after the election is held. In summary, the requirement of a balloting condition implies the dictum that "all politics is local" because of the correspondence of local government structure and any generalization of ballot forms to organize alternatives for the purposes of individual vote choice. As a consequence, it is what is voted on that structures the equilibrium of what is voted for.

Conditions for Simple Majority Rule

This analysis gives a statement of the four conditions necessary for simple majority rule. Firstly, as suggested, these conditions involve the construction of a group decision function. Abstracting from any constitutive group(s), the requirements on the function are that it is 1) single-valued and 2) (well) defined. The former requires a correspondence relation and the latter a measure space that is well, if not completely, ordered.

Proposition 1.0 Group Theory and (Social) Decision Function, $D = f(D_1, D_2, ..., D_N)$.

Remarks. $D = \Gamma(U_1, U_2, ..., U_N) = \phi[0,1] \equiv C(x) = \mathscr{F}(D_1, D_2,..., D_N) = C[0,1]$. $\Gamma(U_1, U_2, ..., U_N)$ is a manifold, and $\mathscr{F}(D_1, D_2, ..., D_N)$ a filter.

Traditional definitions of May's conditions two through four provide the basis for a statement of these conditions and an analysis of the proof of the theorem that SMR is a unique equilibrium.

Proposition 2.0 Anonymity, $D = f(P_1, P_2, ..., P_N)$, where $D_i = \{-1,0,+1\}$ and $U_i = [0,1]$.

Remarks. Riker's condition, that $\phi(P_1, P_2, ..., P_N) = \Gamma(U_1, U_2, ..., U_N)$ is a manifold, $D_i = \{-1,0,+1\}$ is a preference profile, and $U_i = C[0,1]$ is an individual choice set.

Proposition 3.0 Neutrality, $f(-D_1,-D_2, ..., -D_N) = - f(D_1, D_2, ..., D_N)$.

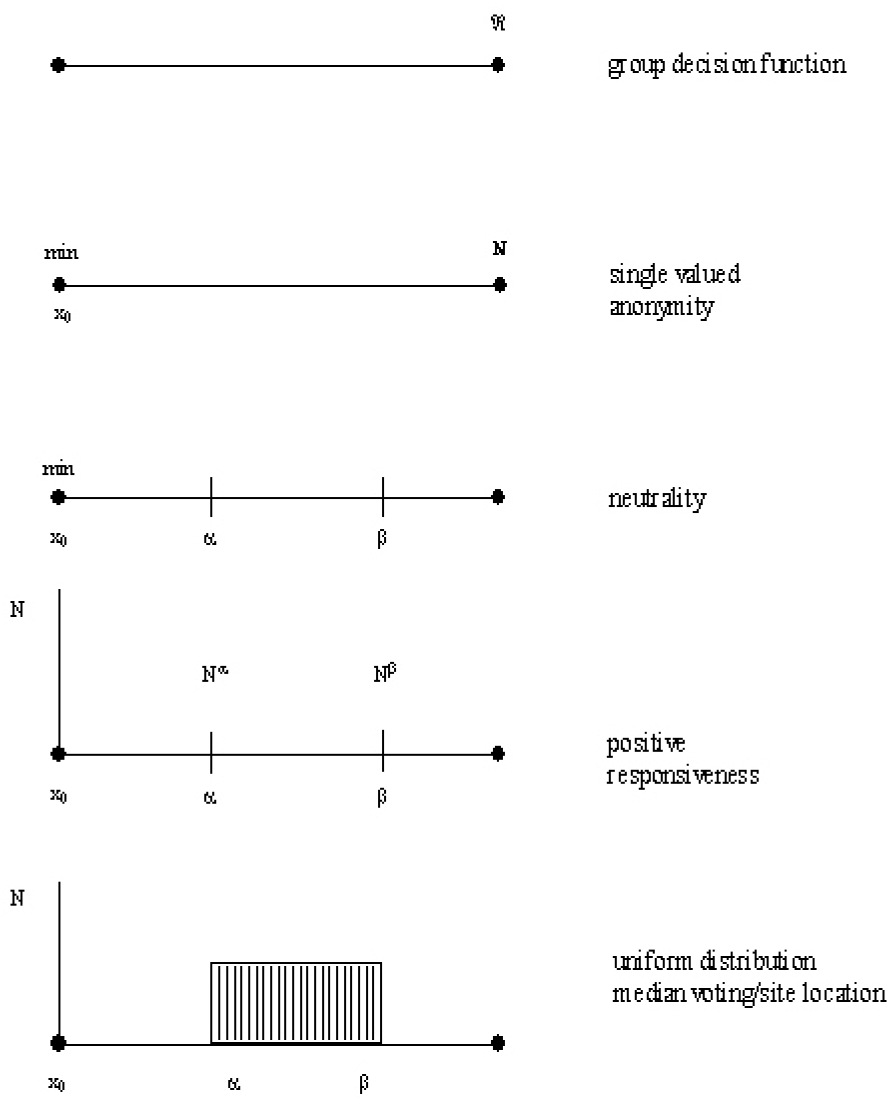
Remarks. $\tau \leftrightarrows f(D_1, D_2, ..., D_N) \rightleftarrows f(D_N, ..., D_2, D_1) \Rightarrow f(-D_1,-D_2, ..., -D_N) = - f(D_1, D_2, ..., D_N)$. 1 to N is a finite integer set that is countably large. If $\tau = \mathcal{H}(\mathbb{N})$ (is Hausdorff), then $\mathcal{H} = \mathscr{L}(D_1, D_2,..., D_N) \Rightarrow \mathscr{L}(D_1 \cap D_2 \cap,..., \cap D_N = \varnothing)$. $\mathscr{L}(D_1, D_2,..., D_N) = f(\ell(D_1), \ell(D_2),..., \ell(D_N)) = \Theta$.

Proposition 4.0 Positive Responsiveness, $D = f(D_1, D_2, ..., D_N) = C(x) = C[0,1]$.

Remarks. $C[0,1]$ is a partition, and $D = \mathscr{F}(D_1, D_2, ..., D_N)$ is a filter. τ is defined for $\mathbb{R}^+$.

Additional results may be obtained from these conditions, however, this analysis will first provide a description of conditions 1 through 4, and then examine the proof of May's Theorem to demonstrate the conditions necessary for a general voting and location equilibrium. In **FIGURE 1.0**, a single dimensional model represents the group theory and decision function with anonymity, neutrality and positive responsiveness constraints.

FIGURE 1 Diagrammatic Exposition of May's theorem



In **FIGURE 1**, the basic components of the model are derived from a decision space, such as those constructed for distributions of votes and locations. The decision space may be single or multi-dimensional, continuous in alternatives or represent a finite integer set in small or large numbers of alternatives. The decision function is defined as $D = f(D_1, D_2, ..., D_N)$ so that collective decisions represent the aggregation of individual decisions. A social decision function refers to rules and procedures used to structure and organize decisions. Voting rules and procedures may also be considered from the generalization of social decision functions. Decisions made in location and distance are yet another social decision functions that aggregate voting and location decisions such as those described by the Tiebout model. Lastly, a group decision function describes the adoption and implementation of decisions by groups that vary in interest and organization. The existence of voting blocs or cartels of voters describe group theory agendas for exerting pressure on individual voters and coalitions of voters to organize and structure decisions.

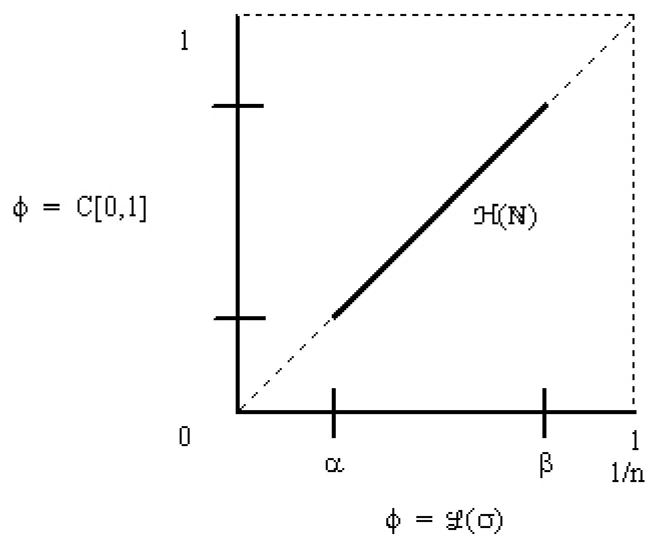
Lemma 1.0 The group decision function is a continuous function.

Proof. **FIGURE 1**. The group decision space and function is defined on $\Re$. $D = f(D_1, D_2, ..., D_N)$, 1. ..., N is an uncountably large number of decisions. $\mathscr{F}(D_1, D_2,..., D_N)$ is a filter for partitioning the decision space. Agenda correspondence, $\mathscr{E} = \mathscr{F}(\nu_1, \nu_2, ..., \nu_N)$. $\Gamma(U_1, U_2, ... , U_N)$ is a strategy derived from the continuous distribution of decisions. $\Gamma(U_1, U_2, ... , U_N) = \mathscr{F}(D_1, D_2,..., D_N)$ is a continuous manifold.

Lemma 2.0 A single-valued correspondence forms an upper-hemi continuous set.

Proof. $\phi = C[0,1] = \mathfrak{L}(\sigma)$. $\mathfrak{L}(\sigma) = \mathfrak{L}[\alpha, \beta]$. $\phi = \mathcal{H}(\mathbb{N})$ is a UHC set.

FIGURE 2
Range and Density in m-dimensional space



Proposition 5.1 Vote space $<\nu> = D(P_1, P_2, ..., P_N)$ is a distribution of voter preferences.

Lemma 3.0 The vote space forms a closed set.

Proof. The decision space is closed by individual profile and decision. The choice set is closed by selection of alternatives. The number of alternatives forms a closed set. The distribution of individual profiles, decisions and choices forms a closed set of voting space outcomes.

Profile	Decision	Choice	Profile	Decision	Choice
$D_i = +1$	+1	support	1	reform	$U_i = 1$
$D_i = 0$	0	abstain	0	status quo	$U_i = 0$
$D_i = -1$	-1	oppose	0	status quo	$U_i = 0$

Proposition 5.2 Vote space $<\nu> = D(\nu_1, \nu_2, ..., \nu_N)$ is a distribution of votes.

Lemma 4.0 The vote space forms a bounded set.

Proof. N = finite integer set. The vote space is bounded by the number of voters. The distribution of votes forms a bounded set.

Lemma 5.0 A closed and bounded set forms a compact set.

Theorem 1.0 A general equilibrium exists in the vote space.

Proof. Lemmas 1 - 5.

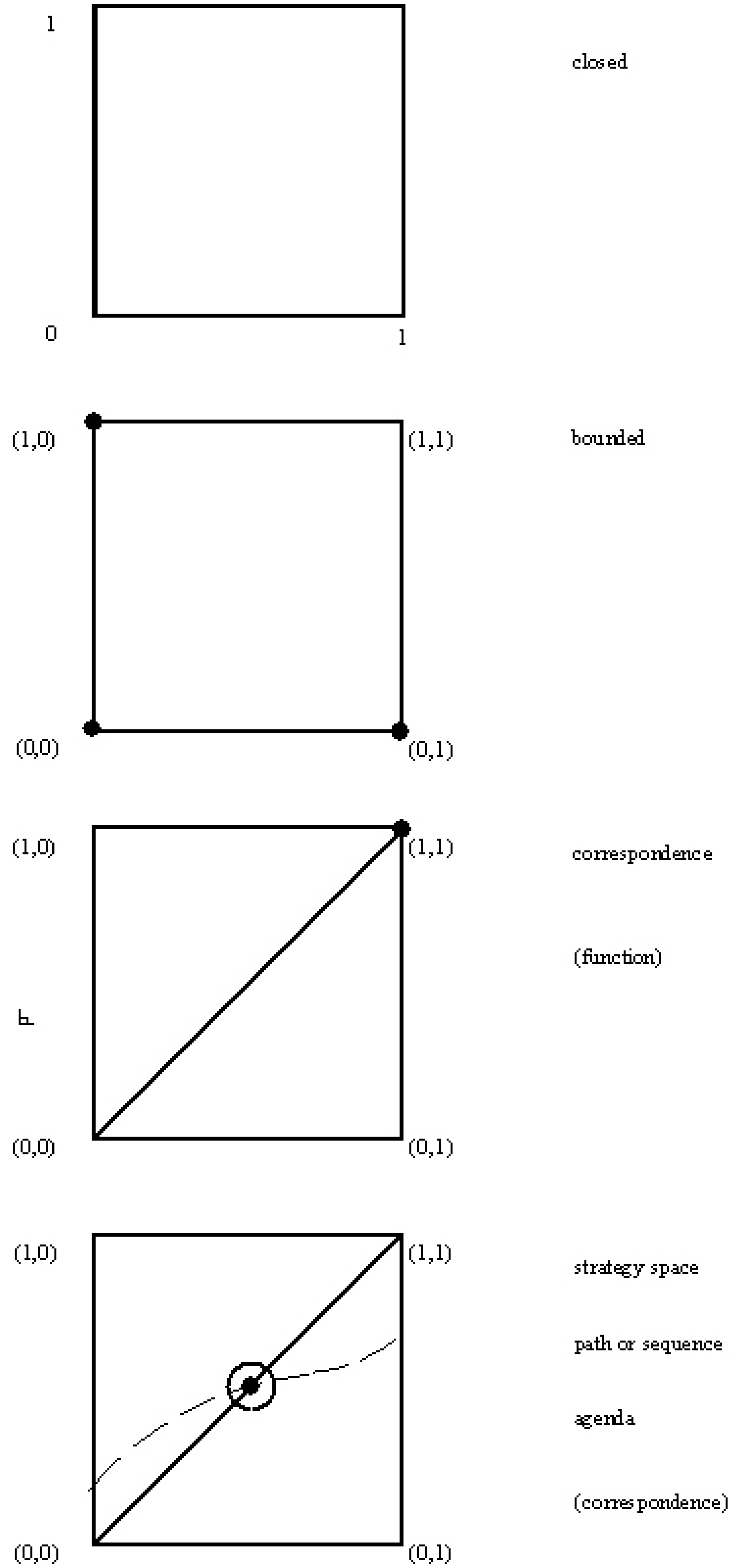
Theorem 2.0 A general equilibrium exists at the voting median site location.

Proof. **FIGURES 1 & 2**. Rectangular or uniform provision of alternatives. β is the least upper bound. α is the minimum lower bound. $\beta - \alpha = \sigma$-range of alternatives. $½\bullet(\beta - \alpha) = .5\bullet\sigma$. One-half the range is equal to a median division. A midpoint exists for any rectangular or uniform distribution of alternatives. The median exists for any set of voting and location decisions. The distribution of alternatives is a closed $R[0,1] = U[0,1]$ set of alternatives. The distribution of alternatives is bounded by $R[\alpha, \beta] = U[\alpha, \beta]$ and therefore $R[\sigma] = U[\sigma]$. σ is a compact set. Setting $C[0,1] = \mathfrak{L}(\sigma)$, $\phi = \mathcal{H}(\mathbb{N})$. ϕ = single-valued correspondence, forming a UHC set.

Theorem 3.0 A general equilibrium exists equal to simple majority rule.

Proof. **FIGURE 3**. The choice set is a closed decision space. The vote space outcomes form a bounded set. The social decision function is a single-valued (UHC) correspondence. Binary voting agendas select SMR.

FIGURE 3 A Diagrammatic Exposition of an Agenda Setter Theorem



Restatement of May's Theorem

Restatements of May's Theorem substitute for the original four conditions. By doing so, this changes the problem from a function with constraints to additional specifications. The idea of maximization of a linear function with (linear) constraints remains as the original formulation of the problem of maximization for choices derived from a distribution of votes in a decision space. The imposition of (linear) constraints on the vote space provides additional traction for equilibrium selection among alternatives.

In the absence of the group theory and social decision function, the problem changes from maximization of a function given constraints, to other problems in voting and location decisions. The substitution of a decisiveness constraint changes the problem to a solution of constraints in the vote space and introduces the possibility of structure-induced voting equilibrium through the adoption and implementation of voting rules and procedures. Given a symmetric distribution of votes, a voting median will exist and maximization may produce, under certain assumptions, the voting median as an equilibrium. In a single dimension, a normal distribution of votes guarantees the existence of a voting median even though spatial competition among voting alternatives and individual location decisions may not generate an equilibrium at the voting median site location. Even so, vote maximization produces a voting median and median site location in a single dimensional normal distribution. The bivariate normal distribution generates a voting median and median site location in a circular distribution of votes in two dimensions. In two dimensions, vote maximization implies selection of a centroid given a minimum radius of error equal to the standard error estimate of the bivariate average or mean. Any general circular distribution of votes contains a larger radius of vote margin or swing than the bivariate normal.

The use of the decisiveness constraint requires the specification of voting rules and procedures. The constraint may be imposed on the distribution of votes, limiting the vote space outcomes to those involving simple majority rule. This result is consistent with maximization of cumulative symmetric distributions of votes in order to produce a voting median and median site location outcome in voting and location decisions. When the constraint is imposed on the distribution of voter preferences, this serves to filter the selection of alternatives and therefore may determine which alternatives are feasible from among those contained in a vote choice set. As a result, decisiveness implies restrictions on voter preferences to guarantee the existence of a median in voting and location decisions. In summary, decisiveness constrains the vote space by partitioning voter preferences into separable clusters or groups and by limiting voting outcomes to within a minimum radius range of the median vote and location decision.

Proposition 6.0 Decisiveness is defined as $D_i = \{-1, 0, +1\}$.

Definition 1.0 Number of voters is defined as the size of the electorate, $N = \{1, ..., n\}$.

Lemma 6.0 $N = \{1, ..., n\}$ is a finite integer, closed and bounded set.

Lemma 7.0 The distribution of votes is a finite integer, closed and bounded set.

Proof. Vote space $<\nu> = D(\nu_1, \nu_2, ..., \nu_N)$ is a distribution of votes. $D(\nu_1, \nu_2, ..., \nu_N)$ is a finite integer, closed and bounded set, $\{1, ..., n\}$.

Definition 2.1 Simple majority rule, odd number of voters: $q = (N + 1) / 2$.

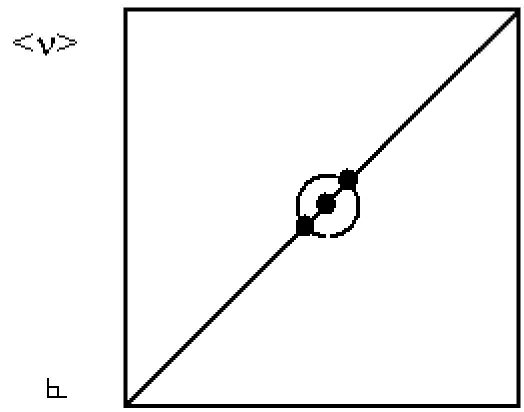
Definition 2.2 Simple majority rule, even number of voters: $q = (N / 2) + 1$.

Definition 3.0 N = size of the legislature (or committee).

Proposition 7.1 Simple majority rule constraint, odd number of voters: $p = (N + 1) / 2$.

Proposition 7.2 Simple majority rule constraint, even number of voters: $p = (N / 2) + 1$.

FIGURE 4 Simple Majority Rule



Proposition 8.1 (Riker) Minimal winning or support coalition, $p = .50 + 1/N$.

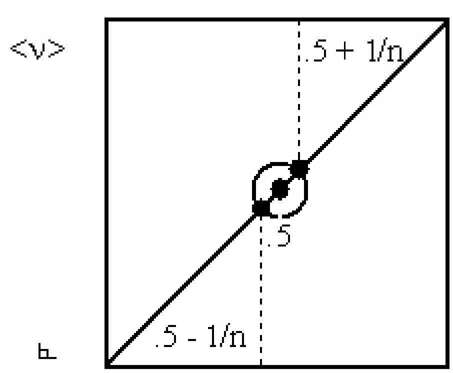
Proposition 8.2 (Rae) Maximal opposition or losing coalition, $p = .50 - 1/N$.

Theorem 4.0 In a finite integer set of voters, there exist $2/N$ decisive votes.

Proof. $p = .50 + 1/N \Rightarrow D_i = 1$. $p = .50 \Rightarrow D_i = 0$. $p = .50 - 1/N \Rightarrow D_i = -1$.

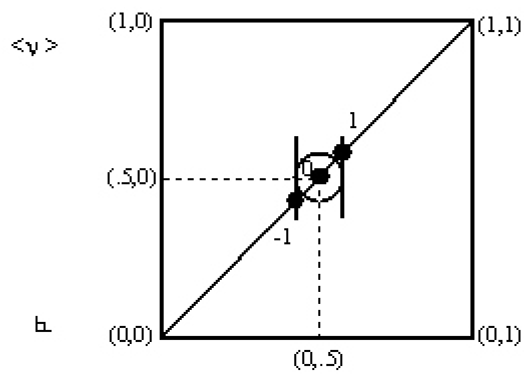
Decisiveness $\equiv$ $D_i = \{-1, 0, +1\}$. $(.50 + 1/N) - (.50 - 1/N) = 2/N$.

FIGURE 5 Decisive Votes



Proposition 9.0 The decisiveness constraint on the vote space.

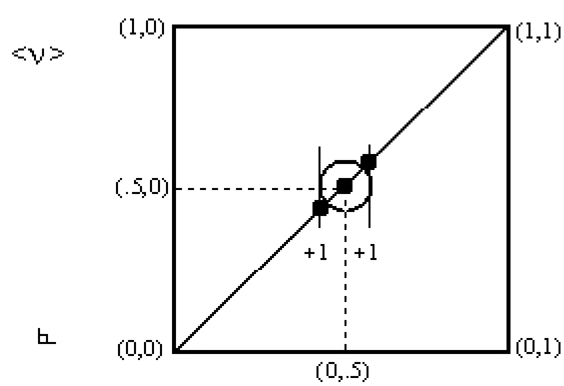
FIGURE 6 Decisiveness Constraint



Proposition 10.1 (Anonymity) $D = f(D_1, D_2, ..., D_N)$, $N = \{1, ..., n\}$.

Proof. Vote space <ν> = $D(\nu_1, \nu_2, ..., \nu_N)$ is a distribution of votes. <ν> is an anonymous distribution of votes = D(1, 1, ..., 1), such that neither the rank order assignment of individual votes, $N = \{1, ..., n\}$, nor the temporal order of voting, $T = \{t_1, ..., t_n\}$ is relevant or statistically related to the distribution of votes.

FIGURE 7 Anonymity Constraint



Proposition 10.2 (Equality) $D = f(1/D_1, 1/D_2, ..., 1/D_N)$, $N = \{1, ..., n\}$.

Proof. Vote space $<\nu> = D(\nu_1, \nu_2, ..., \nu_N)$ is a distribution of votes. $<\nu>$ is an equal distribution of votes $= D[(1/N), (1/N), ..., (1/N)]$. Equality implies one person, one vote.

Proposition 10.3 (Differentiatedness) $D = f[(\omega_1/D_1), (\omega_2/D_2), ..., (\omega_N/D_N)]$, $N = \{1, ..., n\}$.

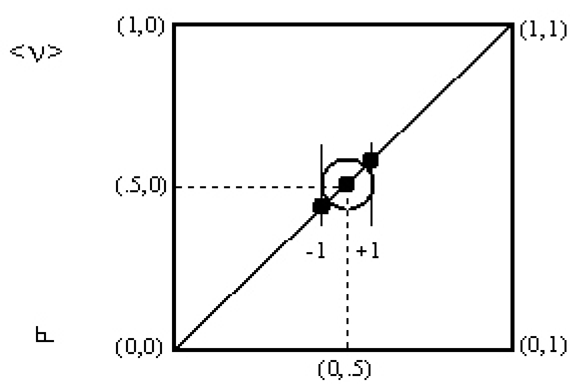
Proof. Vote space $<\nu> = D(\nu_1, \nu_2, ..., \nu_N)$ is a distribution of votes. $<\omega>$ is a weighted distribution of votes $= D[(\omega_1/N), (\omega_2/N), ..., (\omega_N/N)]$.

Lemma 8.0 (Weighted voting scheme) $<\omega> = [\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, ..., w_m; q, N]$, with $m = \{1, ..., m\}$ equal to the vote weights.

Proof. Vote space $<\nu> = D(\nu_1, \nu_2, ..., \nu_N)$ is a distribution of votes. $<\omega>$ is a weighted distribution of votes $= D[(\omega/N), (\omega/N), ..., (\omega/N)]$. A quota method is used to adopt and implement a q-rule for selection among alternatives. N equals the size of the electorate. N is a finite integer set, a closed and bounded set, and therefore the vote space is a compact set.

Proposition 11.0 (Neutrality) $f(-D_1,-D_2, ... , -D_N) = - f(D_1, D_2, ... , D_N)$.

FIGURE 8 Neutrality Constraint

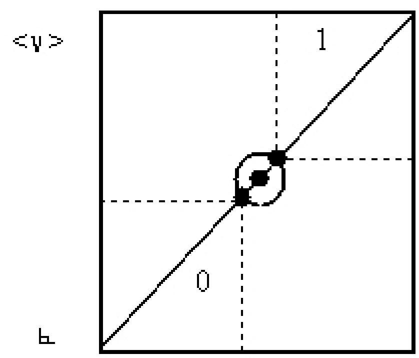


Lemma 9.0 The neutrality constraint is equal to a homothetic function.

Proof. $f(-D_1,-D_2, \ldots , -D_N) = - f(D_1, D_2, \ldots , D_N)$. Positive homogeneity, $\lambda\mathscr{F} = \mathscr{F}(\lambda\nu_1, \lambda\nu_2, \ldots, \lambda\nu_N)$. Given $\lambda\mathscr{F} = \mathscr{F}(\lambda\nu_1, \lambda\nu_2, \ldots, \lambda\nu_N)$, $\mathscr{F}$ is a homothetic function.

Proposition 12.0 (Positive Responsiveness) $D = f(D_1, D_2, \ldots, D_N) = C[0,1]$.

FIGURE 9 Positive Responsiveness



Proposition 13.1 Undifferentiatedness implies unweighted votes.

Proposition 13.2 Undifferentiatedness implies no weighted voting.solution.

Proposition 13.3 Undifferentiatedness implies equality by rectangular or uniform distribution of votes.

Proposition 13.4 Undifferentiatedness implies anonymity in the voting space.

Lemma 10.0 (Undifferentiatedness) $D = f[(\omega/D_1), (\omega/D_2), \ldots, (\omega/D_N)]$, $N = \{1, \ldots, n\}$.

Proof. **Propositions 10.1, 10.2 & 10.3**. **Lemma 8.0**. **Propositions 13.1, 13.2, 136.3 & 13.4**.

Theorem 5.0 (May's Theorem) Decisiveness, anonymity, neutrality and positive responsiveness constraints necessarily and sufficiently imply simple majority rule.

Proof. Given decisiveness (**FIGURES 5 & 6**), anonymity (**FIGURE 7**), neutrality (**FIGURE 8**), and positive responsiveness (**FIGURE 9**) the voting equilibrium equals simple majority rule (**FIGURE 4**). The intersection of the constraint sets equals an even division plus one vote (or .5 + 1/N). This point exists for a finite integer set of voters. The point is located above and to the right of an even or equal division voting outcome. The point equals .50 + 1/N. Given Rae's constraint, equal to simple majority rule (**FIGURE 4**), voting equilibrium exists above and below an even or equal division. The intersection of Rae's constraint, SMR, and the four conditions of decisiveness (**FIGURES 5 & 6**), anonymity (**FIGURE 7**), neutrality (**FIGURE 8**), and positive responsiveness (**FIGURE 9**) equals even division plus one vote. This point establishes a voting outcome equal to .50 + 1/N. The result of the intersection of the voting rule constraint and the four constraint conditions produces a voting equilibrium equal to simple majority rule.

Theorem 6.0 (Rae's Theorem) The simple majority voting rule constraint implies decisiveness, anonymity, neutrality and positive responsiveness conditions.

Proof. **Theorem 5.0**.

Theorem 7.0 A social decision function with anonymity, neutrality and positive responsiveness constraints implies a simple majority rule voting equilibrium.

Proof. **Theorems 3.0, 5.0 & 6.0**.

Theorem 8.0 Group theory and a social decision function that is a single-valued correspondence guarantees the existence of a general voting equilibrium.

Proof. **Proposition 1.0**. **Lemmas 1.0 & 2.0**. **Theorem 3.0**.

Theorem 9.0 A group decision function that is a single-valued correspondence, with anonymity, neutrality and positive responsiveness constraints implies simple majority rule is the unique voting equilibrium.

Proof. The constraints on the decision function are for a single-valued correspondence (**FIGURE 2.0**), anonymity (**FIGURE 7**), neutrality (**FIGURE 8**), and positive responsiveness (**FIGURE 9**). The decision space is described in **FIGURE 1.0**. The simple majority rule constraint (**FIGURE 4.0**) determines the attainment of a voting equilibrium.

The results demonstrate the relationships between May's conditions and simple majority rule. The voting equilibrium established reveals the importance of linear programming with constraints in the analysis. The use of a decision function with constraints guarantees the existence of an equilibrium in the vote space. The inclusion of a simple majority rule voting constraint proves the construction of May's Theorem and generalizes the Theorem to the adoption and implementation of voting rules and procedures. By doing so, this produces choice selection and decision rule maximization in a voting space.

Voting Rules and Procedures

The adoption and implementation of voting rules and procedures generates a sorting and selection of voting alternatives. As a result, there is a structured-induced filtration of alternatives by voting and location decisions. Once again, the use of a decision space produces a closed, bounded, and therefore compact set of alternatives. In this voting space, the results demonstrate a voting equilibrium exists equal to simple majority rule. Given these basic results, the findings suggest a model consisting of maximization of a decision function subject to constraints generalizes to an analysis of voting rules and procedures. This analysis confirms the implications of the constraint conditions and extends the results on simple majority rule to all voting rules and procedures and any structures to organize alternatives for a vote, a binary choice, or agenda sequences.

Besides ascertaining what is feasible, the strategic manipulability of voting rules and procedures implies a structure to voting by organization of alternatives. The failure of pure majority rule results in voting cycles among three or more alternatives in two or more dimensions. By point-to-point tracing of votes, random agendas may be constructed that diverge from an initial point, by drifting and cycling throughout an m-dimensional space derived from voter preferences. In absence of pure majority rule equilibrium, such as a voting or an electoral core, the results suggest random agendas and random walk sequences produce a lack of structure to voting and the possibility of chaotic dynamics in the organization of alternatives. At best, the question of whether to vote on such matters, and if so, the timing of ordering and quorum resolution become critical to voting decisions. As a consequence, the failure of simple majority rule implies a centipede game of search using binary agenda sequences to provide structure.

The existence of empirical minority rule is yet another explanation for the analysis of voting rules and procedures. In cases of minority rule governments, the formation of electoral coalitions becomes of paramount interest to the failure of pure majority rule. In these settings, the edict to "form coalitions" substitutes for the determination of "finding majorities." In this search for any majority, there may be frequent adoption and implementation of voting rules and procedures. Under minority rule, the strategic manipulation of voter registration, the use of plural voting, and the establishment of independent candidates, partisan organization, and constituency representation plans provided a structure for generating unstable minority rule governments.

Beyond the strict majoritarianism of pure majority rule, deliberative democratic methods emphasize the selection of alternatives derived from organized discussion and structured voting. The voting may be informal, in terms of a voice vote, a show of hands or individuals standing up and making a location decision for a seat or position. In committee settings, this may involve strict adherence to Parliamentary procedure and therefore scheduled and apportioned time for discussion and structured voting decisions. A voting agenda is determined ahead of time producing a deliberation with focus on searching for consensus alternatives. By voting on consensus alternatives, deliberative methods imply requirements for adoption and implementation of Paretian alternatives consistent with unanimity rule, consensus votes and greater than a simple majority or super-majority rule voting procedures. The pursuit of a consensus voting equilibrium, such as a 64% majority, suggests deliberative democratic methods produce higher average, super-majority rule voting outcomes, with less variance and therefore greater stability than methods of (simple) majority rule decision.

Both methods of majority rule decision and deliberative democracy imply adoption and implementation of distributions of votes increasing in the vote space. This property is logically consistent with requirements for the conditions of positive responsiveness, positive association, monotonicity and what this study defines as a functional form specification of the voting rule and procedure. Responsiveness stipulates a choice-based selection of alternatives generated for a range of voting outcomes. Association produces a correlation of the distribution of votes with outcomes in the voting space dimension. Monotonicity implies selection of a functional form for representing the relationship between a distribution of votes and outcomes in the voting space. Lastly, model specification is possible to determine the functional form of the relationship between any distribution of votes and outcomes in the voting space. These four conditions describe constraints on the voting space varying by the continuity of the voting rule and procedural correspondence with the distribution of votes.

Proposition 14.1 (Positive responsiveness in the vote space). A positively responsive function represented by selection in the decision space.

Proposition 14.2 (Positive association in the vote space) Positive association defined as linear correlation with the distribution of votes.

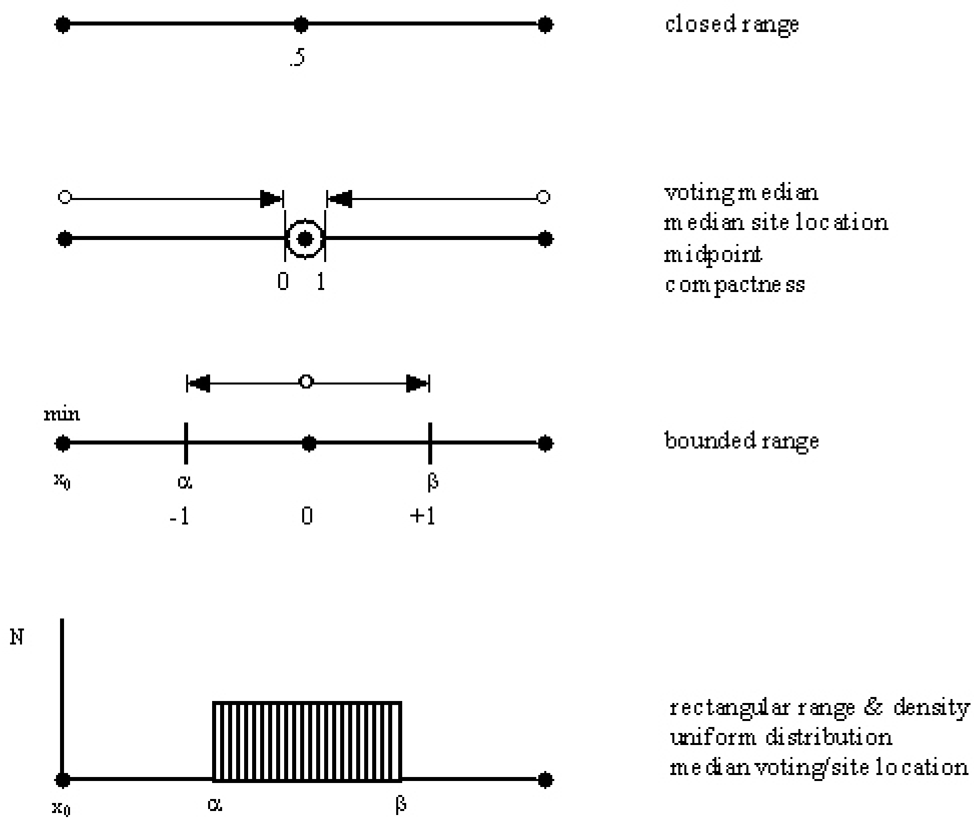
Proposition 14.3 (Monotonicity in the vote space) Monotonicity defined by model selection of functional form and linear correlation with the distribution of votes.

Proposition 14.4 (Functional form in the vote space) Model specification of the functional form explaining the properties of the distribution of votes.

Lemma 11.0 Positive responsiveness satisfies the decisiveness constraint.

Proof.

FIGURE 10 Positive Responsiveness

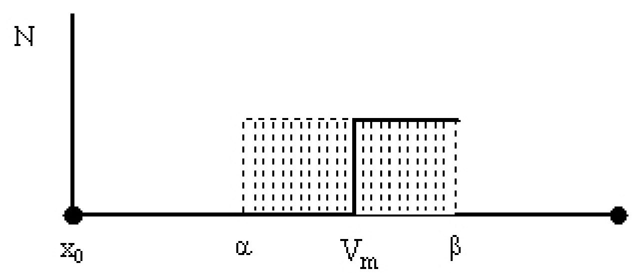


Profile	Decisiveness	Alternative	Outcome	Choice
D	1	β	Reform	1
	0		Status Quo	0
	-1	α	Status Quo	0

Theorem 10.0 Positive responsiveness constrains the vote space at a critical point.

Proof.

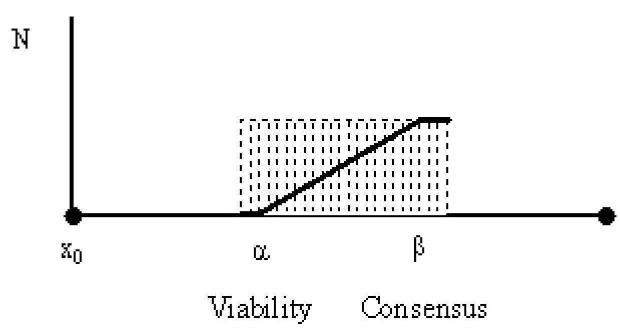
FIGURE 11 Positive Responsiveness Constraint



Lemma 12.0 Positive responsiveness guarantees the existence of a range of voting rule and procedural constraints.

Proof.

FIGURE 12 Positive Responsiveness
Voting Rule and Procedural Constraint



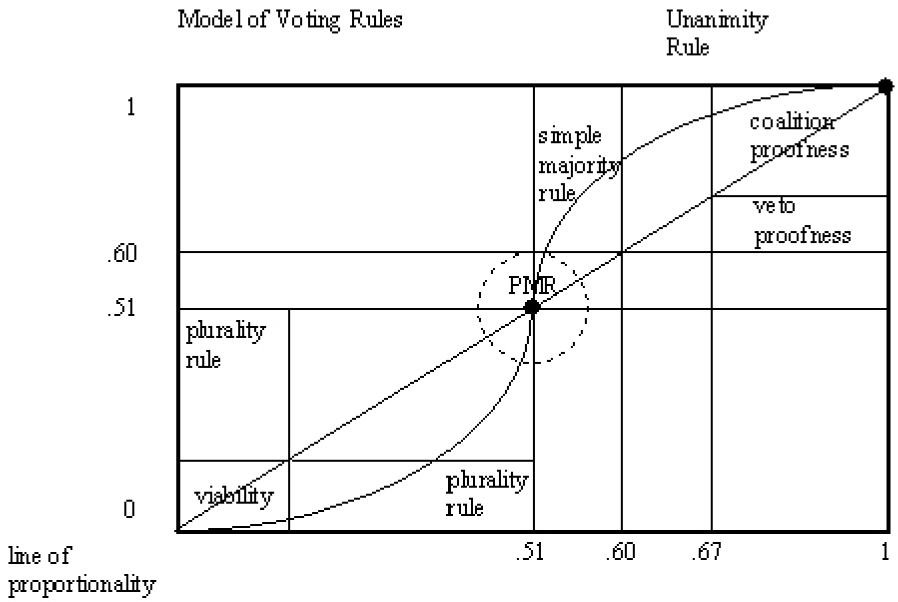
Lemma 13.0 Positive responsiveness satisfies a range of voting rule and procedural constraints.

Proof. **Lemmas 11 & 12**.

Theorem 11.0 Positive responsiveness corresponds with voting rule and procedural constraints.

Proof.

FIGURE 13
Linear Proportionality Filter



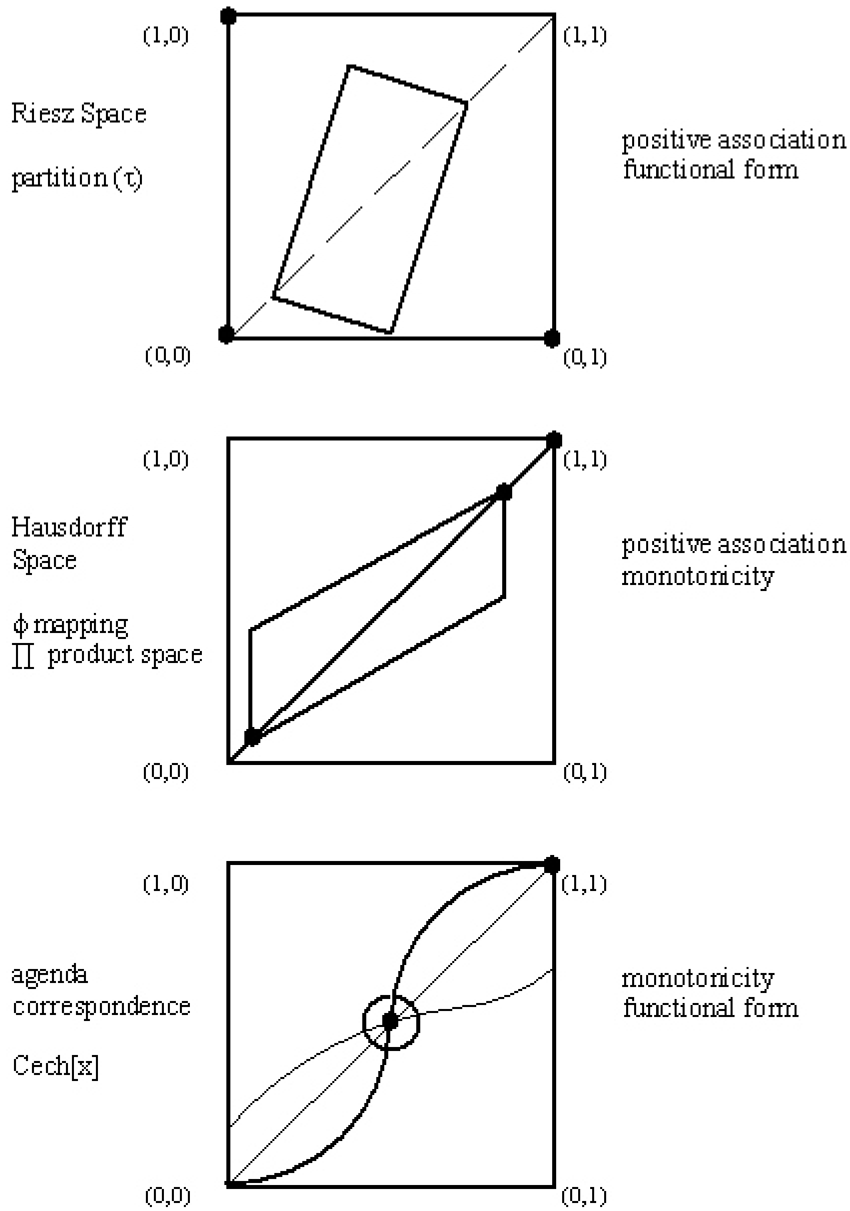
Lemma 14.0 Positive association satisfies a range of voting rule and procedural constraints.

Lemma 15.0 Monotonicity satisfies a range of voting rule and procedural constraints.

Theorem 12.0 Positive Association, Monotonicity and Model Specification of Functional Form satisfy a range of voting rule and procedural constraints.

Proof.

FIGURE 14
Structure of the Agenda Setter Model



Ballot Form and Election Administration

The balloting condition implies a vote decision space constructed by either the allocation of a right to vote or to take a vote in order to make a social or group decision. The vote decision space consists of social or group choices of alternatives. The vote space may contain either a distribution of voter preferences or a distribution of votes derived from casting ballots. Voter identification may be required for balloting in both electoral and committee settings. In an electorate, voter registration is used to identify voters eligible to cast ballots. In a committee setting, the use of a schedule and quorum votes is used to select the timing for roll-call votes. As a result, balloting involves a preference revelation and a vote. In an electorate, individual vote choices may be anonymous, but the turnout decision to vote is known information. In a committee, individual representatives know whether and how other members of the legislature voted by roll call and recorded votes.

Definition 4.0 Balloting is the tallying and counting of votes.

Definition 5.0 Election returns are the reported aggregate or cumulative counts of votes for the ballot alternatives.

Definition 6.0 Vote validation is a sampling of ballots cast to verify election returns.

Definition 7.0 Recounting describes additional tallies or counts of votes required because of the closeness of the vote or because of voting irregularities producing systematic errors in the tallying and counting of the ballots.

Definition 8.0 A sample ballot form is an agenda structure, format or codex for voting on alternatives.

Proposition 15.1 A office block ballot structures votes by elected seat or position.

Proposition 15.2 A partisan ballot structures votes by partisan designation of the alternatives.

Proposition 15.3 Constituency voting is by geographic district listed on the ballot.

Proposition 15.4 Party list voting is by strait partisan ticket listed on the ballot.

Proposition 15.5 A mixed representation plan has both constituency and party list voting.

Proposition 15.6 A short ballot describes voting on a small number (or sequence) of alternatives.

Proposition 15.7 A long ballot describes voting on a large number (or sequence) of alternatives.

Proposition 15.8 A primary is a nomination election with election administration by state and local government.

Proposition 15.9 A caucus is a nomination election with election administration by political party organization and precinct, county and Congressional District..

Theorem 13.0 A ballot form is a voting agenda to structure and organize alternatives.

Proof. **Propositions 15.1-15.9** construct a ballot design equal to a codex. The codex conforms to a tournament structure, centipede game of search and selection. This structure is equivalent to binomial search and selection, using voting agendas.

Proposition 16.1 A regular ballot is cast by precinct district.

Proposition 16.2 An absentee ballot is mailed in to election administration and is counted separately from votes cast by precinct.

Proposition 17.1 A balloting condition is not required for the adoption and implementation of group theory and social decision functions.

Proposition 17.2 A balloting condition produces a single-valued correspondence between the distribution of votes and election returns.

Proposition 17.3 (Bush v. Gore) A deficient ballot format may produce multi-valued correspondence between the distribution of votes and election returns.

Proposition 17.4 Balloting is a necessary condition for anonymity of votes/voters.

Proposition 17.5 Balloting is a necessary condition for neutrality of voting alternatives.

Proposition 17.6 Balloting is a necessary condition for positive responsiveness in the vote space.

Proposition 17.7 Balloting is a necessary condition for decisiveness in the vote space.

Proposition 17.8 Balloting is a necessary condition for simple majority rule.

Proposition 17.9 A ballot form is not required for simple majority rule or methods of majority decision.

Proposition 17.10 Voting and election administration are required for simple majority rule and methods of majority decision.

Theorem 14.0 Balloting is a necessary condition for the adoption and implementation of voting rules and procedural constraints.

Proof. **Propositions 17.1-17.10**. A secret ballot is required for anonymity of votes/voters. The ballot form guarantees the neutrality of voting alternatives. Voting or casting ballots generates positive responsiveness in the vote space. Simple majorities and decisive pivot or swing votes are determined through counts and tallies derived from the distribution of votes.

Caucus and Primary Election Voting Rules and Procedures

A caucus voting rule and procedure was used for nomination elections at the state and national levels for elective offices. The caucus method has been replaced with nomination by partisan conventions and primary elections. In the Presidential nomination process, the few remaining caucus states retain partisan organization of election administration. The precinct caucus meetings establish vote support for the candidates to determine viable vote shares. Once determined, these vote shares are used to allocate delegates by states and state districts to the national Democrat and Republican Party Conventions.

Caucus methods traditionally did not have paper ballots. Instead, location voting was used to identify vote support for the candidates by forming coalitions at the precinct meetings. The coalitions form, by seat and position, for each candidate. Zero vote candidates receive no caucus support. In what is referred to as the initial alignment, voters locate the group or coalition of votes in support of their preferred candidate. In practice, the campaigns have volunteers in each precinct and these individuals post a sign above a location in the precinct meeting area for where vote supports are supposed to locate. As a result, the individuals make a location decision first, deciding where they prefer to locate by seat or position. Choosing the group or coalition location involves a choice of known individual voters and is therefore inconsistent with the anonymity condition. Caucus voting is therefore location voting, producing a combination of location and voting decisions by searching for groups and forming coalitions.

In recent years, voting rules and procedures have been imposed by the national political party conventions on states using the caucus method. In many states, paper ballot forms consisting of scan-tron answer sheets are handed out to registered voters as they enter the precinct meetings. The voters attend the precinct meetings, where the campaigns discuss the candidates and issues. The voters cast ballots and turn them in to the partisan officials of the precinct. In some state's precinct caucuses, this is the end of the voting process. The ballots are turned in and counted by partisan administration. In other states, viability rules have been established for a minimum vote for a candidates to be defined as a feasible alternative. In some states, the viability rule is 15% of the first alignment, or initial casting of ballot forms. In these states, candidates holding below the 15% voting rule procedurally constrains the caucus voting to engage in additional rounds of voting. The campaigns continue and a second round of voting occurs with ballots turned in to establish a second tally or count of votes for the candidates. If a candidate falls below the viability threshold, the voting rule and procedure eliminates the candidate as a feasible alternative. This produces a third round of voting. As a result, the caucus method, even using paper ballots, is a voting by elimination, for selecting consensus alternatives.

Under the traditional caucus method, there were no paper ballots. The caucuses begin with groups and voters form coalitions by location decision only. The selection of a location is therefore the vote cast. At any individual location, all of the voters support the same candidate and therefore Tiebout sorting occurs that produces homogeneous groups. Any vote at the location is unanimous and therefore only the number of voters at each location is used to tally or count the number of votes for each candidate. Because voters can determine whether their candidate is viable from any coalition alignment, voter's that prefer a lower ranked, non-viable, candidate must make a relocation decision. By doing so, they choose another coalition location to support their second or third ranked candidate.

In summary, caucus voting is based on a group decision function and location voting decisions to form coalitions for alternatives. Caucus voting also produces more complicated vote outcomes because of the elimination of alternatives and therefore any additional rounds of voting. In a primary election, voters cast a paper ballot that is tallied or counted *once* as an initial alignment of votes cast. The election returns do not indicate whether any of the candidates are non-viable or fall below any voting rule and procedural constraint eliminating the candidate as a feasible alternative. Caucus voting begins with an initial coalition alignment of voters by location of seat or position at the precinct meetings. As a mechanism, the caucus method produces an alignment and realignments of coalition structures through individual location (voting) decisions. For this reason, realignments are continuous during the precinct meetings and this includes the decision to leave a precinct meeting, if the individual's preferred candidates are not viable. As a result, the total number of voters is reduced as individual candidates are eliminated as being non-viable, lower ranked alternatives. The numbers of voters supporting the viable candidates also varies by round of voting, as individuals make relocation decisions by changing their coalition alignment. As the precinct meetings progress, what is sometimes described as a game of musical chairs produces a final alignment of groups and coalition support for candidates. The vote shares produced in the final alignment are the vote shares reported as election returns and used for the purposes of delegate allocation.

In an effort to improve reporting of election returns, caucus states were required to have paper ballots and to report initial and final coalition alignment votes. This requirement implies no individual relocation decisions, including leaving the precinct meetings, until an initial tally or count is completed. Instead of making location decisions, individuals were instructed to make a single location decision and then remain at this location until a count was completed of the initial alignment. On this basis, the rule and procedural changes attempted to change something that is inherent to the mechanism design of caucus voting, by imposing a static location decision on what are group dynamics in a committee setting. By the time the final alignments were derived from coalition adjustments, the numbers of votes were substantially reduced and there were substantive changes in the relative numbers of votes for each of the viable candidates.

In an effort to improve voter participation, absentee ballots were collected in some states holding precinct caucus meetings. Under the traditional caucus method, only the individuals that attend the precinct meetings vote for the candidates. The use of paper ballots at the meetings is somewhat irrelevant to this consideration, except that by introducing a friction to the precinct meetings, by time lapse between counts of votes, the waiting times between votes could potentially increase the rate of attrition in the total number of voters in attendance at the precinct meetings. In the states using absentee ballots, the delay between rounds of voting was increased as precinct chairs and partisan organizations recalculated precinct meeting votes and absentee votes. The absentee ballot forms were designed for the purposes of ranked choice voting, so that individuals that voted by absentee ballot were required to indicate their first, second, third, ..., etc. ranked candidates. Individuals were not required to rank all of the candidates and many absentee voters had first and second preferences for non-viable candidates. This did not set their votes aside, but instead, required those counting the ballots to assign their vote to their top ranked candidate remaining among the feasible alternatives. As a result, absentee voters frequently voted for their second & third ranked candidates derived from their preference orderings.

The amount of time it takes to count an initial alignment or to reassign absentee votes based on their ranked choices may be substantial in terms of the organization of precinct meetings. Since individuals attend to participate, these voting rule and procedural constraints impose conditions that may be logically inconsistent with the mechanism design of caucus voting. As people sit around, waiting for additional instructions to participate, there are incentives to move the nomination process forward by making individual decisions to join other coalitions, to become de-aligned from a group, and potentially to exit the precinct meetings. As the number of absentee ballots increases and far exceeds the number attending precinct meetings, this further erodes the organization of precinct meetings by the political parties and partisan campaign organizations. The ballot form required to incorporate absentee votes reveals that caucus voting is a form of ranked choice voting. Beyond the mechanism design, the caucus method generates a rank ordering of candidates that many interpret as more meaningful than the election returns reported by caucus or primary election. As an alternative, the primary election only reports an initial coalition alignment and does not impose viability rules or allow for relocation decisions in additional rounds of voting. Instead, primary elections report more fractional vote shares, failing to eliminate infeasible candidates as alternatives. Early voting also produces some differences in the ranked choices of voters because of the exit of candidates from the campaign. In some states, there were substantial differences in the vote shares conditional upon the timing of voting, with very long duration vote periods adding to the complexity of counting vote shares cast one month prior to election day to those ballot cast on election day. As candidates exit the campaign, and endorse a candidate among the viable alternatives, some states allow individuals to "revote" by revoking their initial ballot and casting a second ballot.

Whether revoting can be accomplished by ranked choice voting is another matter. The timing of delays to caucus voting is increasing from the imposition of a rule and procedural constraint to count and report the initial alignment, final alignment and delegate allocation share, to reassign absentee ballots by ranked choice, and to allow voters to revoke their initial ballots to be able to cast a more accurate or non-wasted vote for a candidate among the remaining feasible alternatives. Even so, this points out the importance of ballot form and a balloting condition to voter preferences and counting the distribution of votes.

This analysis implies the strategic manipulability of methods of majority rule by voting rules and procedures such as the inclusion of a viability rule for determining feasible alternatives. The inability of primary electorate's to form a majority produces electoral returns with plurality rule winning alternatives. Most importantly, the plurality rule winning alternative may hold a low vote share and the distribution of votes, for a large number of candidate alternatives, may result in a fraction distribution of vote shares for the candidates. As a result, neither the plurality winning candidate nor the rank order of candidates produces legitimacy or credibility derived from the election returns. Because the purpose of Presidential nomination events are to select state delegates to the national party conventions, the issue concerns the transformation of fractional vote shares into proportional or winner-take-all delegate allocation. The failure of pure majority rule with three or more candidates guarantees that it is possible for a primary election to generate a fractional plurality winner and rank ordering of the top ranked candidates. This divided vote problem may be considered less important with proportional allocation of state delegates. Even so, the amount of fractionalization may produce cause for concern among the viable alternatives and any infeasible candidates that have not exited the campaign.

The failure of pure majority rule implies using methods of majority decision with voting rules and procedural constraints. Generally speaking, this failure may also provide a rationale for using minority rule procedures, such as the viability constraint, or arguing for deliberative methods to produce a search and selection of consensus alternatives. This can be accomplished by use of simple majority rule with a tournament structure, binary voting agendas, and therefore a search and selection centipede game for the organization of alternatives through multiple rounds of voting by paired comparisons. By requiring multiple votes, there is an elimination of alternatives, a revelation of a rank ordering of the alternatives, and an agenda sequence of paired comparisons. The agenda sequence does not guarantee the selection of a voting median and the failure of pure majority rule implies the nonexistence of a voting median for three or more alternatives in multi-dimensional space. As a consequence, having a right to vote on specific matters and election administration of individual voting decisions may not produce a simple majority rule winning outcome equal to the median voter preference for the alternatives.

This study suggests the use of location decisions provides another electoral mechanism. The use of sorting and selection methods guarantees the existence of individual location voting decisions, homogeneous groups, strategic coalition formation, and a median site location. The voting and location equilibrium produced is a general equilibrium derived from individual location voting decisions, coalition formation and adjustment decisions. As a mechanism, caucus voting guarantees the existence of a general equilibrium and produces a consensus alternative and a rank ordering of the alternatives. On this basis, caucus voting should be considered a deliberative method, a voting rule and procedure adopted and implemented to determine consensus among alternatives, and a limited form of ranked choice voting.

Other reforms suggest changes to ballot formats with the assumption that changing the ballot structure changes the organization of voting alternatives. By doing so, structure-induced voting equilibria exist conditional upon the voting rule and procedural constraints adopted and implemented. Among the most comprehensive is a change to a mixed representation system, electing representatives derived from both constituency districts and party list selection of candidates. In the United States, mixed representation plans are enacted by charter to establish both citywide at-large districts and city ward-constituency districts. These mixed representation plans vary in the distribution of vote power allocated to at-large and ward-district seats or positions. Additional reforms consider the creation of varying delegation size multi-member districts to replace excessive division of local jurisdictions into single member districts. This limited form of weighted voting generates varying ballot structures to elect single representatives from those elected in multi-member districts. In three member districts, the ballot form may provide for partisan contestation by one party only or both political parties, producing combinations from three to six partisan candidate alternatives on the ballot.

Going in the opposite direction, there have been reform efforts to eliminate straight party ticket voting from ballot forms. The option to select a political party, to vote for all the candidates nominated by the political party, has been eliminated by voting rule and procedure in several states. This reform reduces the potential for a multi-dimensional vote among all the partisan elected offices on a ballot. By design, the ballot form only consists of office block comparisons of candidates nominated by the major and minor political parties. This voting rule and procedure reduces the ballot structure to voting dimension-by-dimension, one office at a time in agenda sequence by seat or position.

The balloting condition implies a citizen's sovereignty condition or a right to vote on alternatives. As a result, the balloting condition establishes a vote for candidates and issues or matters of policy requiring a social or group decision. The ballot format maps individual voter preferences into the vote space of election returns. By mapping the distribution of votes through the social or group decision function, the ballot provides individual voter's a structure for the organization of alternatives for the purposes of holding an election. In the absence of a ballot format, or no paper ballot at all, are caucus voting methods that incorporate the location decisions of individual voters to produce election returns.

In summary, the vote decision space is both structured and manipulable by the adoption and implementation of voting rules and procedures. Voting rules and procedures determine constraints on the mapping of the distribution votes, and therefore voter preferences, into a rank ordering of the alternatives in election returns. By method of majority decision, structure-induced voting equilibrium exist by long versus short ballots, tournament structure, agenda design, election calenders, rounds of voting and binomial (centipede) games of search and selection.